



Cthulhu's Wormhole: A Tale of Many Legs

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Background & Motivation

- Einstein's theory of gravity, General Relativity (GR), describes how **spacetime** and matter interact
- Spacetime can be curved (e.g. sphere), allowing **wormholes** to connect 2 otherwise disconnected regions

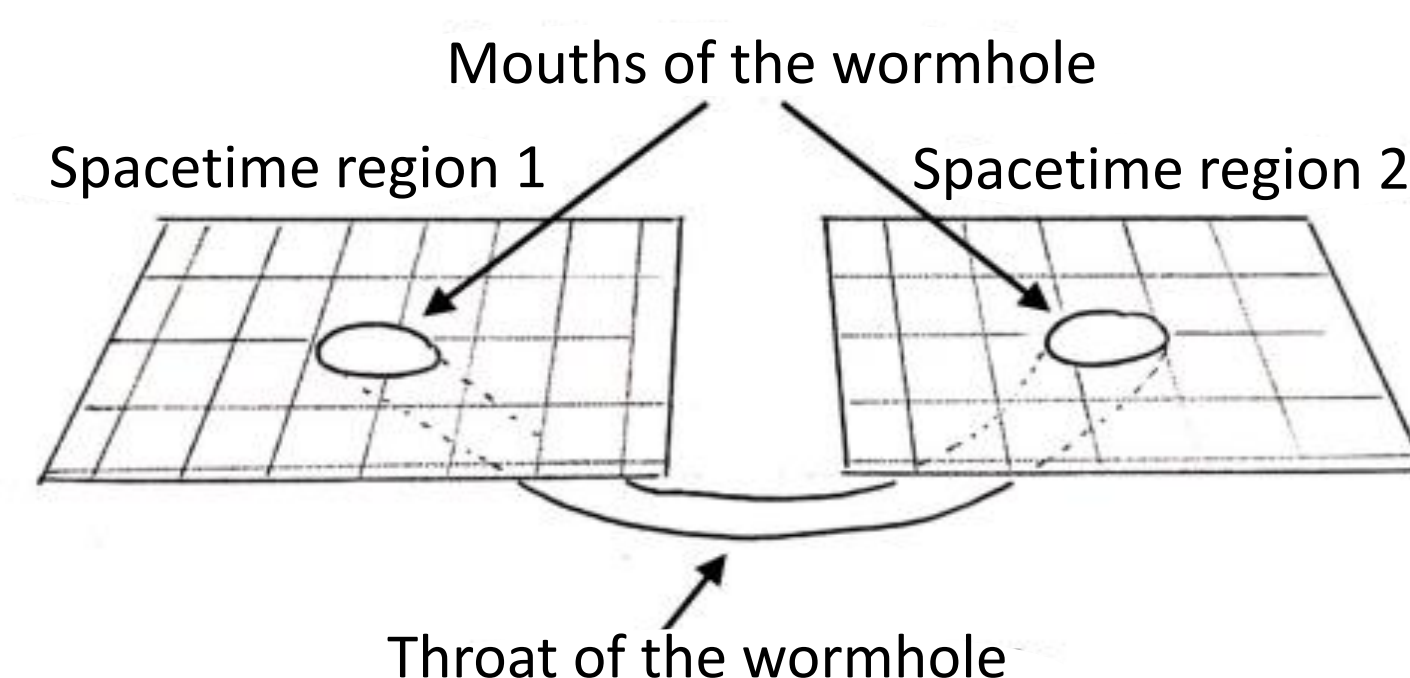


Figure 1: Illustration of a wormhole in 2-dimensional spacetime

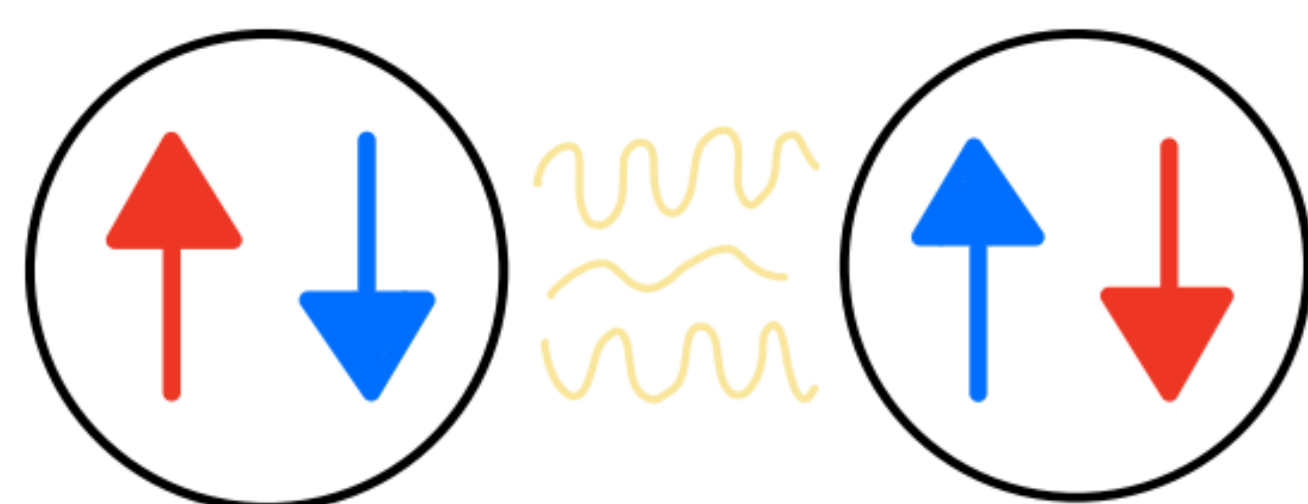


Fig. 2: Maximally entangled pair of, say, electrons

- Entanglement:** in quantum theory, a collection of particles cannot be described independently of other collections with which it's entangled
- Occurs when collections interact

A powerful idea in theoretical physics has been that **entanglement may help build geometry (spacetime and its properties)**

- Anti-de Sitter (AdS)** space: vacuum (no matter) solution to Einstein's field equations with negative cosmological constant (negative curvature):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

- We will work with a constant time slice of AdS_3 (3-dimensional AdS)
- Geodesics** here are circular arcs



Fig. 3: Visualization of AdS_2 using Poincaré disk (diverging metric at boundary)

- Conformal Field Theory (CFT):** a quantum field theory invariant under changes of length, with local angles being preserved

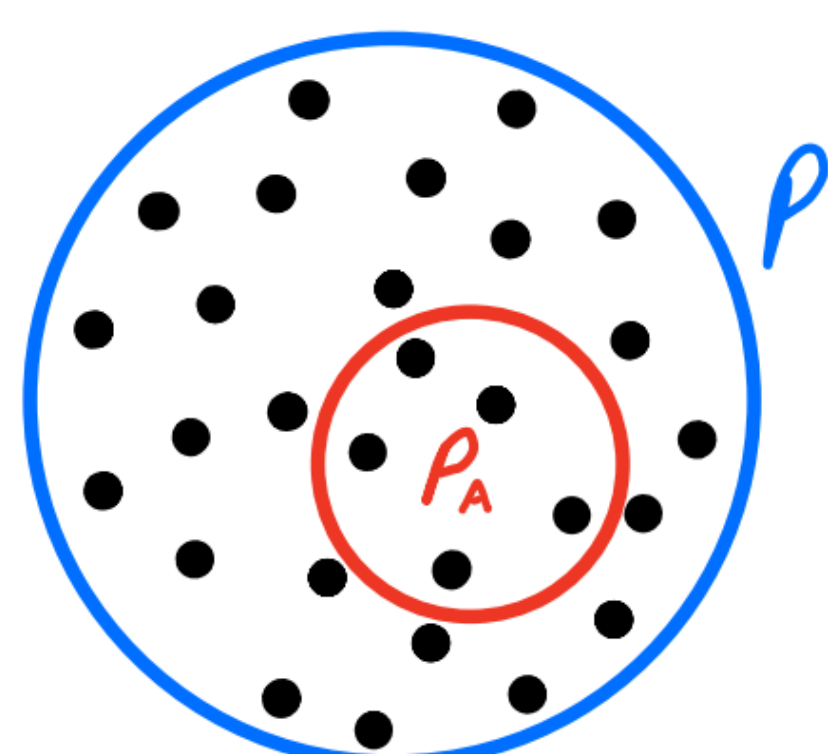


Fig. 4: Partial trace of a system

- Entanglement entropy:** how entangled a subsystem is with the rest of the system:

$$S(\rho_A) = -\text{tr}[\rho_A \log \rho_A]$$

We want to understand more about how entanglement produces geometry by studying the entanglement structure of wormhole geometries

Methods & Results

- AdS/CFT Correspondence:**

- D-dimensional AdS "equals" (D-1) dimensional CFT

- RT Formula:**

$$S(A) = \frac{1}{4G_N} \text{area}(\gamma(A))$$

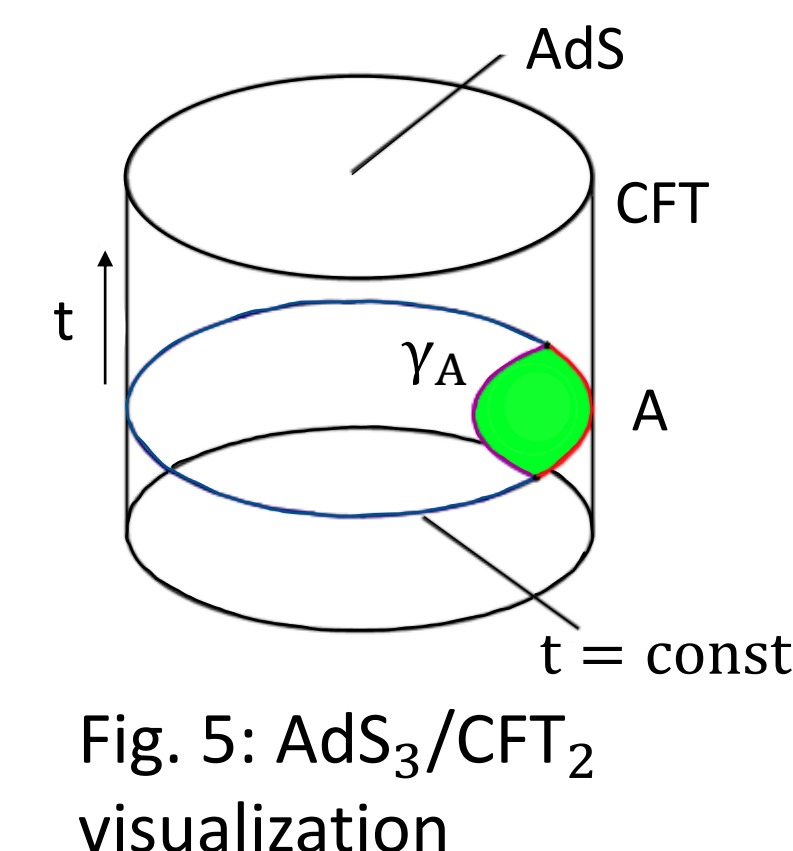


Fig. 5: AdS_3/CFT_2 visualization

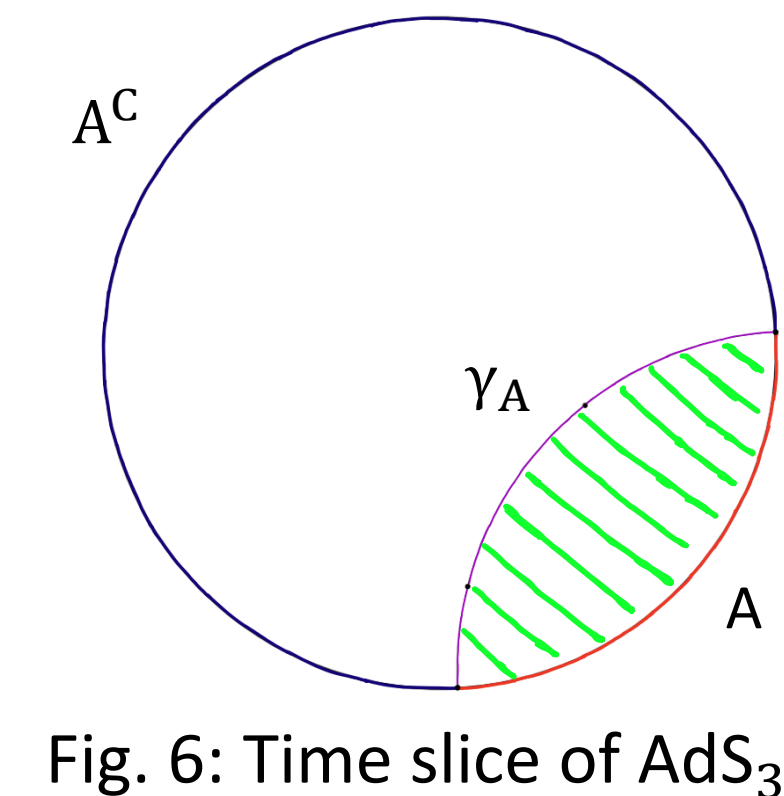


Fig. 6: Time slice of AdS_3

- 2- and 3-Boundary Wormholes in AdS_3/CFT_2 :**

- H is the (event) horizon length
- Completely determines geometry

- Identify** same color geodesics

- Mutual Information**

- A measure of correlations between A and B, quantifying how much you learn about A from measuring B:

$$I(A; B) = S(A) + S(B) - S(AB)$$

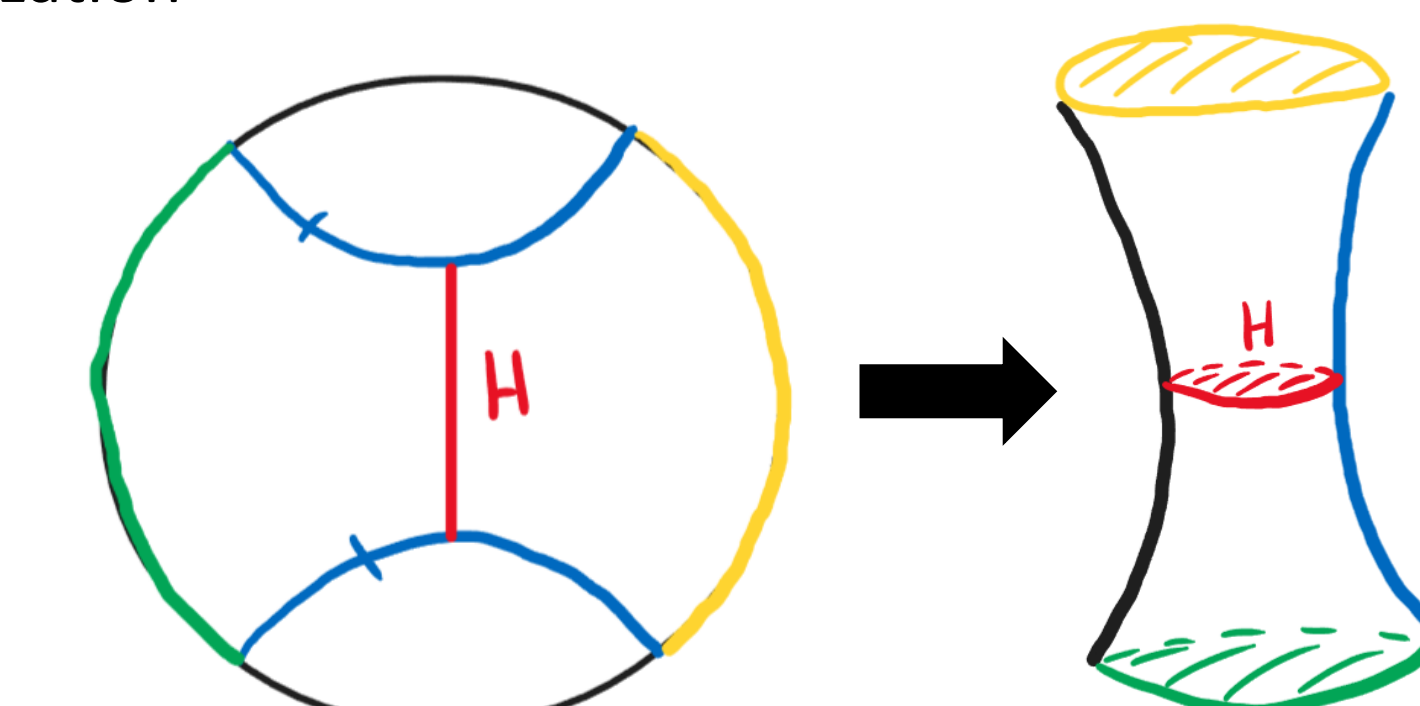


Fig. 7: Identification of BTZ wormhole (genus-0)

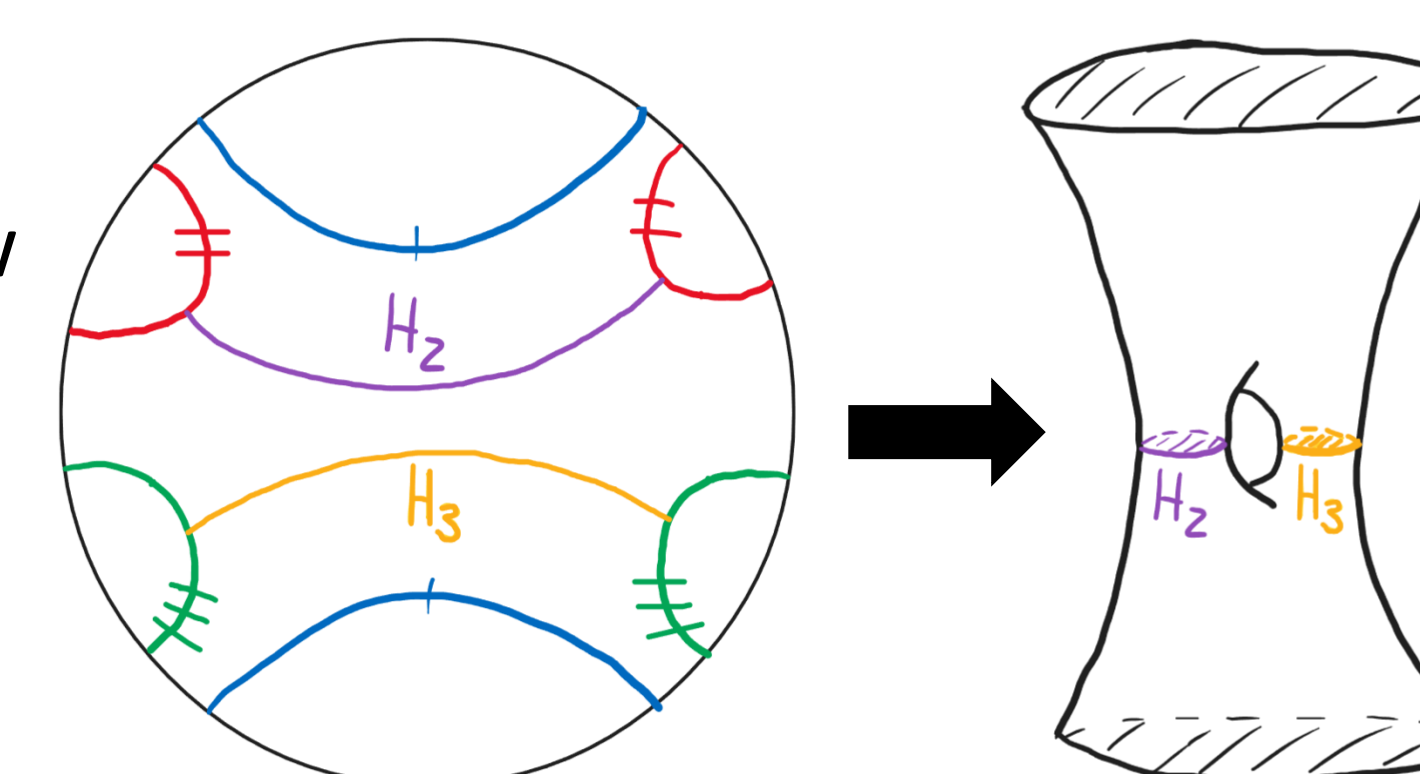


Fig. 8: Identification of genus-1 wormhole

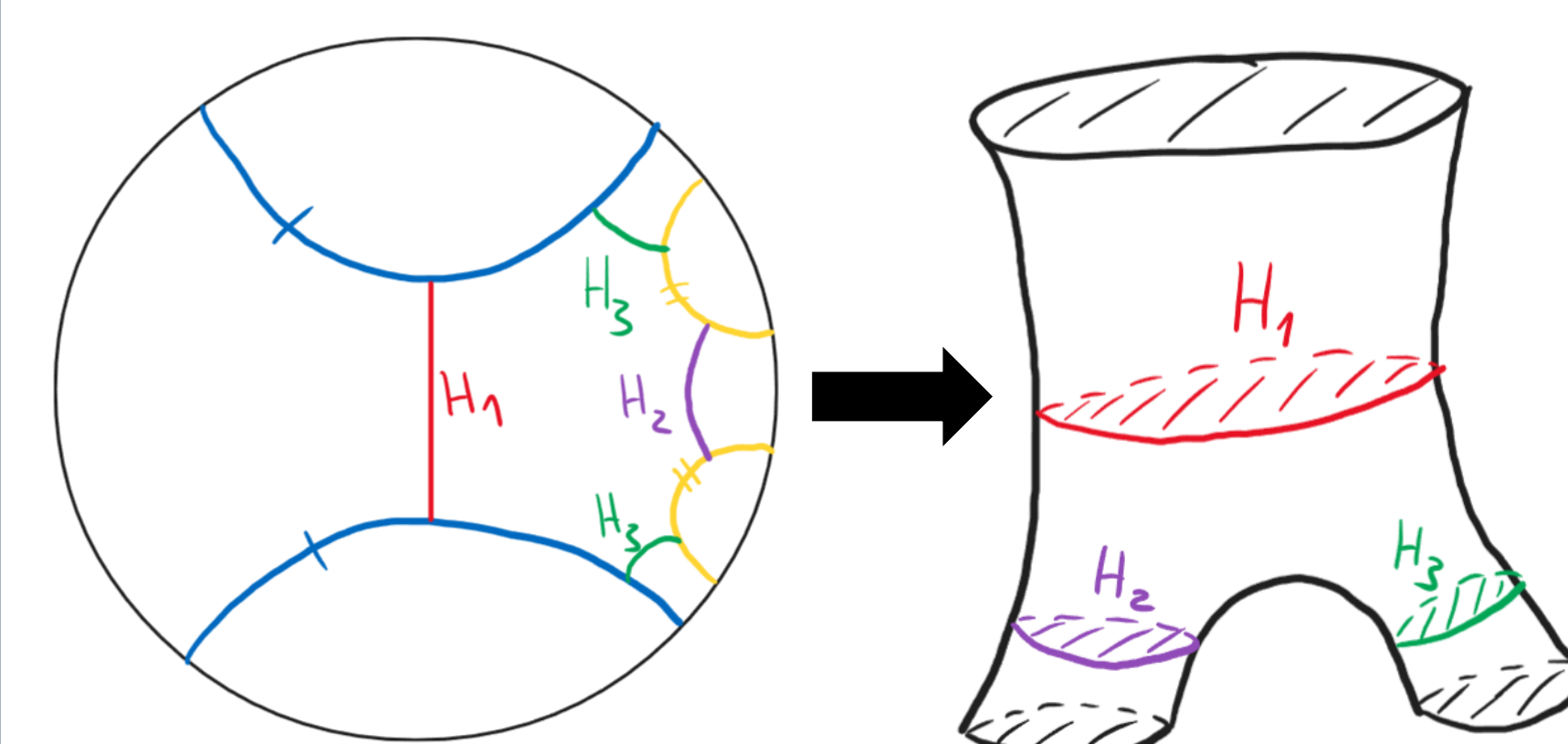


Fig. 9: Identification of 3 boundary wormhole (pair of pants)

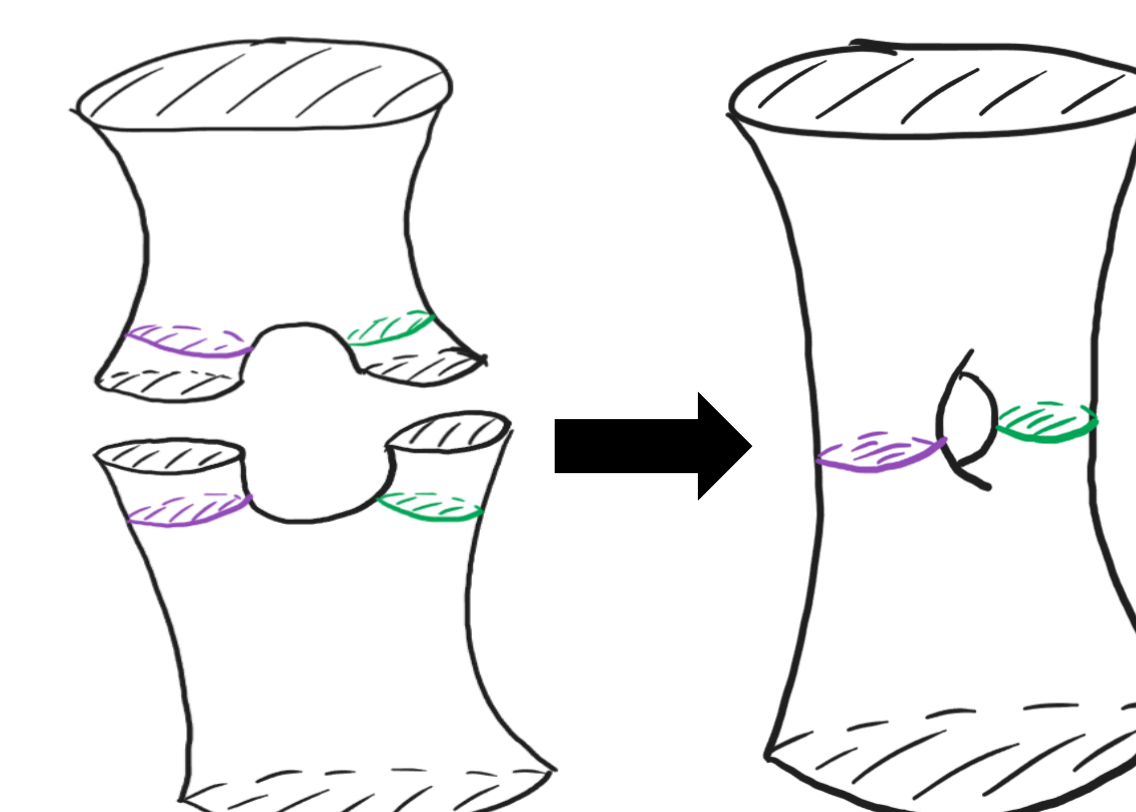


Fig. 10: Canonical purification (gluing together)

- At phase transition,** a measurement of mutual information of the regions below should differentiate between genus-0 and genus-1

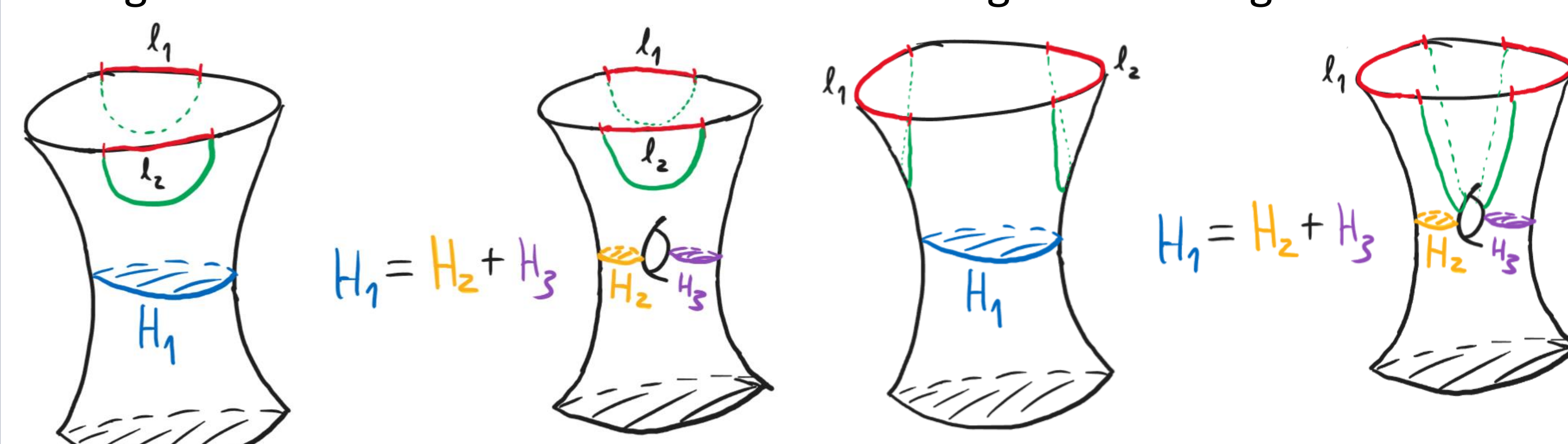


Fig. 11: Example measurement with same lengths of green lines

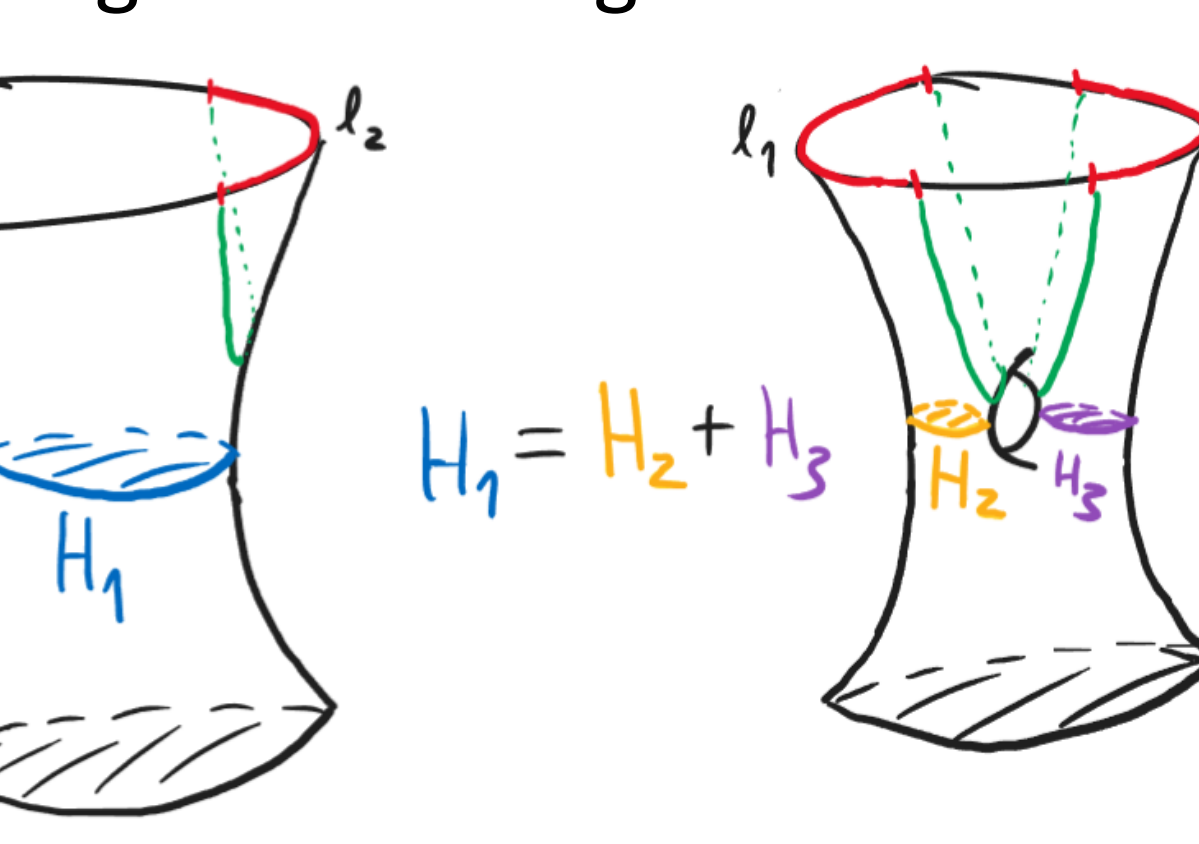


Fig. 12: Example measurement with different lengths of green lines

Conclusions

- Self-entanglement on wormhole mouths plays major role in producing 2 wormholes at phase transition with different geometries
- Next steps:
 - Write efficient algorithm to compute the deviation in, say, mutual information between subregions on the mouths of genus-0 and genus-1 wormholes
 - Consider more general genus-n wormhole (Cthulhu's wormhole)...

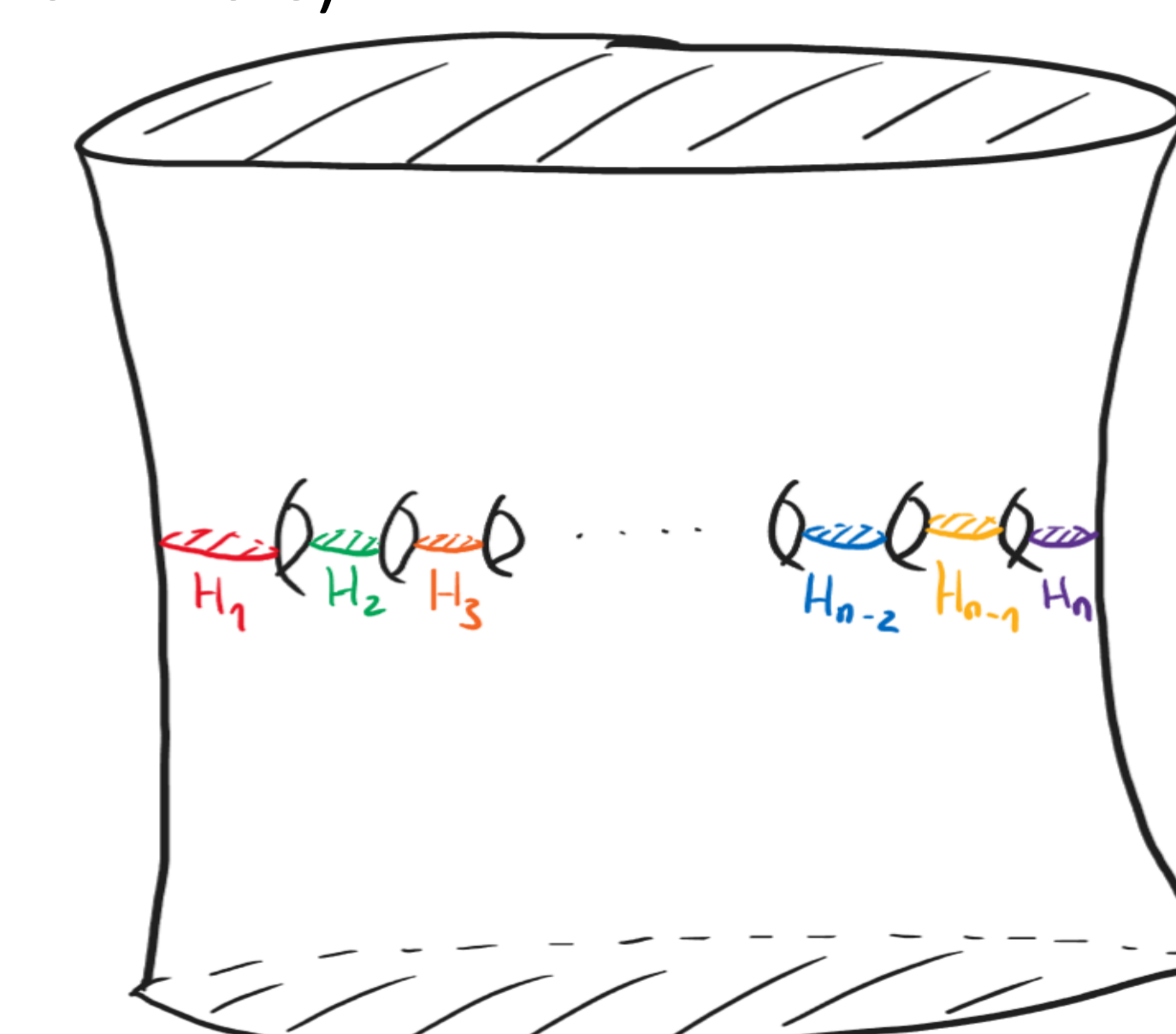


Fig. 13: Wormhole of genus n (i.e. n holes in the bulk)

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